Expecting the Unexpected: Professional Growth through the Highlights and Challenges of a Problem Solving Classroom

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Two teachers, good friends and colleagues, and yet with quite different approaches to the teaching of mathematics, exploring what was for them "new territory" (pedagogically and mathematically) as they taught a problem based unit of work for the first time. Using classroom observations and post-observational interviews, this study described the challenges of the teaching of the unit through teacher identification of "critical incidents" that were encountered. This paper tells the story of professional growth through highlights and challenges, and explores issues of support provided by colleagues and the researcher.

The Call for Change

In recent years, around the world there have been calls for reform in the teaching of mathematics (e.g., Australian Education Council, 1990; National Council of Teachers of Mathematics, 1989). Many factors are influencing such pressures for change including new technology, the needs of a 21st century workforce, new insights from research into how children learn and changing perceptions of what it means to know and do mathematics. Current calls for reform present a contrasting picture of the mathematics classroom from that of traditional mathematics instruction. In the United States, the *Curriculum and Evaluation Standards* (NCTM, 1989, pp. 5-6) presented five general goals for all students:

- that they learn to value mathematics;
- that they become confident in their ability to do mathematics;
- that they become mathematical problem solvers;
- that they learn to communicate mathematically; and
- that they learn to reason mathematically.

The implementation of these goals requires a different classroom environment from that which has come to be called "traditional practice". *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990, pp. 18-20) reflects similar goals, with a strong emphasis on the student being actively involved ("students should experience the reward of arriving at a solution through their own initiative and persistence and not simply through imitation"), with the teacher as a facilitator or guide of learning ("helping students learn to seek imaginative solutions to problems in constructive ways") (AEC, 1990, p. 21).

Though teaching can never be viewed as a completely predictable act, the role of the teacher in the traditional mathematics classroom is to structure the lesson in such a way as to minimise unexpected events. However, when a teacher is responding to students' thinking, and challenging students to explore and conjecture, it becomes a case of "expecting the unexpected."

Background to the Study

The study described in this paper was undertaken in the United States. It was a case study of two experienced seventh-grade mathematics teachers as they taught an innovative unit of work, the mathematical content of which was angles, similar triangles, and introduction to the tangent ratio embedded in context which included hang gliding, shadows and "blind areas" in shipping. The two teachers (Ms Brennan and Ms Whitten—fictitious names) were colleagues who worked closely together but whose approach to teaching provided a considerable contrast.

The questions that guided the study were:

1. What kinds of critical incidents for the teachers arise during the teaching of a unit of mathematics built around realistic situations?
2. In what contexts do such critical incidents arise?
3. How do teachers respond to these critical incidents and what influences their responses?

The unit of work that provided the mathematical content for this study was one taken from a National Science Foundation curriculum development initiative. *Mathematics in Context: A Connected Curriculum* (Romberg et al., 1991) was a mathematics project for grades 5-8. The stated philosophy underlying the project was the belief that mathematics is fallible, changing, and like any other body of knowledge, the product of human inventiveness (Ernest, 1991).

Instructional units which were built around non-routine problems were initially developed by the Freudenthal Institute at the University of Utrecht in The Netherlands, and "Americanised" as required. These units provided opportunities for the active involvement of the student, both physically and mentally. The mathematics content was challenging and yet Dutch experience showed that the realistic contexts seemed to make the problems accessible to the students (de Lange, 1987; Treffers, 1987). Alternative solutions to problems were encouraged. Although this was a curriculum development project, its purpose was not to produce "teacher-proof" materials, but rather to encourage teachers to experiment with a new style of teaching by providing appropriate curriculum materials and ongoing inservice and within-class support.

Methodology

This study was based on the belief that teaching is a complex activity (Koeler & Grouws, 1992). The decisions teachers make are largely situation-specific and as such need to be looked at in the context in which they occur. The perspective taken by the researcher in this study was one of participant-observer which Erickson (1986, p. 121) described as *interpretive*.

Erickson argued that this perspective is best at answering questions like "What is happening, specifically, in social action that takes place in this particular setting? [and] What do these actions mean to the actors involved in them, at the moment the actions took place?" (p. 121). These are clearly the type of questions being asked in this study.

The research involved a case study of two teachers as they taught a particular *Mathematics in Context* unit to a particular class of students and met together after most lessons for planning and reflection with the researcher. Initial observations and interviews prior to the teaching of the unit provided background information as well as familiarisation with student and teacher routines. Once the teaching of the unit began, information was gathered in the form of field notes, post-observational interviews with the teachers, and audiotaping of lessons.

In total, 58 lessons were observed and 23 observational and two summary interviews were conducted. Interviews provided the opportunity to document critical incidents as identified by the teachers, and also enabled the teachers to discuss the reasons for the teaching and planning decisions made in response to these.

"Critical incidents" were those classroom situations identified by the teachers as memorable and significant—both challenges and highlights. As the task of identification rested solely with the teachers, this process was clearly influenced by their past experience, their teaching style, and their mathematical confidence and competence.

Audiotapes of interviews and some classroom interactions were transcribed. As these critical incidents were noted from the transcripts, the corresponding points in both the field notes and journal were combined with the teachers' comments from the interviews. Together, these formed a "story" of each incident. As the transcripts were being read and re-read for critical incidents, relevant background information, and general themes were noted. All data, interviews, field notes and journals were electronically "cut and paste" into different areas of interest. These included the critical incidents,

information on background and classroom practices for each of the teachers, general school issues, mathematical challenges, and assessment issues, and these formed a starting point for the writing process.

Critical Incidents

This study attempted to identify and describe those situations that occur in the teaching of a "realistic" unit that were challenging and difficult for the teacher. Although other studies (see, e.g., Ball, 1988) and pilot testing of *Mathematics in Context* units provided anecdotal examples, a more in-depth study of the unexpected situations that arise on a day-to-day basis had the potential to give a clearer picture of what the challenges are in the teaching of this type of unit. These were not isolated incidents but occurred within the context of a particular classroom environment.

The incidents were grouped into broad categories, where the particular focus was on what constituted the "trigger" for the incident. This should not be considered a definitive categorization, the decision of categorization being based on my interpretation, drawing on all the available data relevant to each critical incident, including field notes from classroom observations, classroom audio tapes, interviews transcripts, and journal entries.

The broad categories were:

- Student insightful response.
- Student incorrect response.
- Student affective response.
- Challenge to the teacher's knowledge.
- Challenge to the teacher's beliefs.

There were incidents which involved more than one component of this categorization. For example, an incorrect response from a student may challenge a teacher's knowledge, clearly cutting across at least two categories. Of equal importance with the categorization were the detailed descriptions of the incidents themselves as they represented important and significant classroom incidents for these teachers as they taught an innovative curriculum unit for the first time (see Clarke, 1995).

For both teachers, the insightful responses of students were the most common source of critical incidents. These included incidents where students were conjecturing and posing questions. They were student-initiated solutions of considerable variety and sophistication, which were often highlights, but sometimes challenging in their resolution.

The teachers identified only a small number of situations resulting from students' incorrect solutions, probably because many of the student errors were of the kind commonly experienced in all mathematics classrooms. For example, students misreading the scale on a protractor in Ms Brennan's class was not reported in post-lesson interviews. Both teachers identified situations involving student mistakes where the source of the error was difficult to determine at the time.

Incidents that were triggered by students' affective responses included situations where the students were so engaged in the thinking and context of the problem that lively debate turned into heated discussion about a particular mathematical issue.

Other critical incidents arose because of challenges to the teachers' mathematical understanding. Their prior mathematical experience (both as teacher and student) had been largely the presentation of an algorithm followed by twenty or thirty exercises for which the algorithm was a perfect match. When faced with problems requiring flexible, conceptually-based understanding, they found that their mathematical knowledge was inadequate. The interviews provided a non-threatening environment in which to discuss any difficulties the teachers were having, particularly such mathematical ones.

Ms Brennan's Response to Incidents

The teaching of this unit yielded many unexpected situations for Ms Brennan that were different from the types of incidents associated with traditional mathematics teaching.

> Those are good surprises that come up because you haven't said, "this is how to do it". You never do that, so having them come up with those generalisations, that's good. The discovery part of it. This is real open-ended. So you really don't know what you might end up with. (Interview, Ms Brennan)

This unit was also challenging for the students. "It's harder for them because you're forcing them to have to think" (Interview, Ms Brennan). Ms Brennan viewed this as a positive aspect of the use of this type of unit. "I like it a lot better. I think it makes them think and makes them use what they know".

Ms Brennan showed a willingness throughout the unit to be a risk taker, both pedagogically and mathematically. When discussing in general terms the situations that arose, she was asked if she saw herself as a risk taker:

> I'm probably more of a risk taker than most, I guess. I wasn't before. I would, you know, only do certain things, but this [material] makes it easier to take the risks and do it with the kids. (Interview, Ms Brennan)

Although the unit covered content with which she was not familiar, she was not discouraged. On the contrary it was a positive experience for her:

> You have to love to learn. I think that's one of the things you have to do. If you don't like learning then this isn't what you need to be in. You have to like learning and you learn each time you do it with them. (Interview, Ms Brennan)

The incidents identified by Ms Brennan were not only those that were challenging, but also those that were memorable and affirming. There was a sense of anticipation, almost excitement as she taught this unit, and critical incidents were often viewed very positively, being seen as indicators that there was worthwhile mathematical activity going on.

Ms Whitten's Response to Incidents

Ms Whitten was not quite as comfortable with this type of unit as Ms Brennan. During one interview, she discussed her comfort level with this unit and those like it. She had had a discussion with another teacher at the school who was currently teaching her first unit. They had compared the teaching of this with the teaching of a Social Studies program called "Quest."

> Well it's very much like this in terms of preparation. The materials, the activity level, the grouping and you are always go go. She said it's exhausting. I said that that's really true. . . It's hard on the teacher doing it all the time. (Interview, Ms Whitten)

Ms Whitten found that the teaching of these units was more demanding, both physically and emotionally:

> It is dealing with questions, dealing with more activity. In that way it's demanding of your energy, physically and emotionally. There's the preparation before, the thinking it through, the trying, setting up the materials. It's always clean it up, next class, clean it up, . . .
>
> Making it understandable for that at least 50% of the kids that are going to struggle somewhat. . . . I found they were catching on pretty well but I kept feeling I'm hand-feeding them more than I

> should. I would have liked to [provide] less help and yet then it would go so slowly. . . You know, there's pluses and minuses to helping them along. . . (Interview, Ms Whitten)

The role of the traditional mathematics teacher is to structure each lesson with particular outcomes in mind. Ms Whitten was more comfortable with the pace of a traditional classroom. The thinking and on-the-spot decision making of this new approach were demanding for her:

> The teachable moment, that kind of occurs more. Having a variety of answers, sometimes surprising answers coming back at you. Definitely more thinking situations. (Interview, Ms Whitten)

In summary, Ms Whitten, a thoroughly-prepared, dedicated teacher was a willing participant in the project, but interviews and observations revealed a commitment to traditional practice. The importance placed on students' success which tended to tie her to traditional practice was also an influence in her growing commitment to the need for a change: "seeing them do well and understanding gives the program more validity and I'm willing to put the time into it" (Interview, Ms Whitten).

During the teaching of this unit and our discussions, Ms Whitten came to question aspects of her teaching. Though always reflective of her practice and willing to listen to new ideas, this particular experience had been pivotal. She was more prepared to allow the students to explore the mathematics themselves.

Toward the end of the study, Ms Whitten expressed the view that she may look to take a year off teaching to rethink what she was doing. She did this and at the time of writing I am unsure as to whether she has returned to teaching. The factors in such a decision are complex, but her realisation that different expectations were now required of her was expressed to me as a major concern. During the time of the study, Ms Whitten gained confidence and commitment to this approach. Though it presented a number of challenges, the regular discussions and support of both Ms Brennan and myself seemed to make these manageable. Ms Whitten was an excellent classroom manager who was genuinely concerned for her students and it would be of concern if such expertise was lost to the teaching profession.

The Professional Growth of the Two Teachers

The teachers' responses to the incidents and the subsequent discussion clearly showed a change in their practice and a growth in their understanding of the role of the teacher. It is reasonable to assume that challenges are likely to be present in any classroom where a teacher is "wrestling" with creating this kind of "constructivist" environment. In fact, the data in this study supported the hypothesis that the more the classroom resembles such an environment, the greater the incidence of critical incidents of the kind discussed in this study. This is supported by the work of Shroyer (1981), who suggested that there was a greater frequency of critical incidents in a problem solving classroom than in a traditional one.

Structuring Classroom Discussions and Presentations

The amount of structure varied between teachers, and both too little and too much structure seemed to limit the students' thinking. For example, with the initial presentation of problems there was a need to clarify, but at the same time not to present complete explanations. The challenge was to present a problem in such a way that the students could make a start, while not limiting their thinking by over explanation; that is, clarification without explanation. This may sound an easy distinction, but in practice it was not, as it depended on the problem context, the problem itself, the students' prior experiences, and the teacher's knowledge of the students and the mathematics.

There is no set formula for the amount of structure that is desirable in a given situation. Romagnano (1991) considered this to be an unsolvable dilemma. Nevertheless, it is likely that teachers will become more confident in determining the level of structure, as they acquire more experience with the use of innovative materials.

Student Comfort Level

Another challenge the teaching of this unit presented for one teacher in particular was that of student comfort versus struggle. The emotional well being of the students was very important to this teacher, and she worked very hard at making the classroom environment comfortable and successful. She would tend to react quietly to an incorrect student response so as not to embarrass the student. Students were not generally encouraged to struggle and it is this very struggle, the grappling with mathematical ideas, that reform documents encourage. This represented a contradiction between the practices which this teacher believed allowed students to be comfortable and successful and the intended purpose of the unit.

Issues of student affect were very important to both teachers, though in quite different ways. For one teacher (Ms Whitten), the emphasis was on removing anxiety and providing regular success. For the other teacher (Ms Brennan), it was important that students were motivated and she was more willing to embrace the teaching approaches advocated by the unit's authors because the students were more enthusiastic, and it reinforced what she valued.

Limited Mathematical Understanding

The teachers found the mathematical content of the unit challenging. They were required to understand an innovative unit of work and a consequent range of insightful and incorrect student solutions, for which their previous mathematical background had not necessarily equipped them. Creating an environment of conjecturing and inventing is particularly difficult where teachers feel they are at the limits of their own knowledge of content. As Brophy (1991) argued:

> Where (teachers') knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasize interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)

Teachers' lack of mathematical knowledge led to critical incidents but it also affected the responses to incidents where the teachers were unsure how to proceed. Both teachers relied heavily on the regular discussions with the researcher and each other to clarify their own mathematical understandings. They were able to learn through the process and were encouraged by this, but the mathematical demands of the unit of work remained a constant challenge for them.

It is interesting to reflect however that in Ms Brennan's case, her own struggles with mathematics sometimes provided greater opportunities for the students. There were several occasions when Ms Brennan's doubt about mathematical content led to her opening up the discussion to students' ideas, in the hope of resolution of the difficulty. Despite this, while connected and integrated mathematical knowledge on the part of the teacher is not a sufficient condition for a classroom environment that is of the kind recommended in reform documents, the data from this study lend support to Brophy's argument that it is desirable.

Physical, Mental, and Emotional Demands on the Teacher

The requirements of this type of teaching made great physical, mental, and emotional demands on both teachers, more intense than those of traditional mathematics teaching. Excitement, frustration, uncertainty, and enthusiasm were common emotions felt by the teachers, sometimes all in the one lesson. Ms Whitten, though increasingly realising the value of both the classroom materials and the recommended approach to teaching, found the demands of managing a fast-paced and more boisterous classroom where students were, for example, making models of the Grand Canyon, measuring boats, creating and flying paper aeroplanes, and offering a variety of unexpected solutions to problems, a considerable contrast from her usual, orderly classroom. Ms Whitten took the opportunity occasionally to "catch her breath" by providing review lessons.

Ms Brennan, much more comfortable with the unexpected nature of the interactions and willing to "go with the flow", nevertheless found the work quite intensive. It is very difficult to be interacting, listening in an attentive way, trying to understand students' solutions, and thinking of an appropriate response, many times during a lesson. In a more conventional classroom, the teacher knows the "right" way to do a problem because they were the ones who explained it to the students in the first place.

Some Implications

This study provides illustrations of the kinds of critical incidents that arise in a problem solving classroom as well as considerations that a teacher must balance as she makes spur of the moment decisions. Although each situation is a unique combination of multiple influences, it is hoped that the story of these teachers' struggles will provide insight into this type of classroom, and will therefore be useful for the professional development of mathematics teachers. Particular incidents have the potential to be used to engage teachers in discussion and debate about what it means to know and to do mathematics and how student learning can be enhanced in such a classroom.

Shulman (1991) discusses the use of "cases" as a promising tool to "infuse reality and concreteness into theory-laden courses of teacher preparation and staff development" (p. 28). Such cases represent readable accounts written by teachers of teaching episodes or series of events that represent recurring challenges for teachers. These include the context in which the situations occur as well as the thoughts and feelings of the teachers involved. They provide an opportunity for teachers to reflect in a manner that is difficult in the classroom setting, as "too much is happening too swiftly in the messy world of practice for teachers to think deeply about what they are doing" (p. 29). This study generated similar rich descriptions of classroom situations which, although not teacher narratives, provide insight into the struggles for teachers in a new classroom environment.

The combination of curriculum materials, previous staff development, and support by project staff encouraged the teachers to pursue a different approach to their teaching. However, it is a major demand on teachers to make significant changes to their practices when they may have to teach five classes every day, as well as their other administrative responsibilities. There is little time for the kind of reflection that is needed if they are seriously going to look at the implications of these critical incidents and the units on their practice. One of the positive aspects for the teachers in this study was the opportunity to share with each other and the researcher on a regular basis. Both teachers commented on the value of having a support person to encourage them in what was a difficult task. In this study, the role of the researcher could not easily be separated from that of project support person, as the teachers struggled with both mathematical and pedagogical issues.

Conclusion

When students are encouraged to invent, conjecture, and solve realistic mathematical problems, there will be situations that are unexpected and challenging for the teacher. Avoiding these may lead to a limiting of students' thinking, but confronting them leads to a very demanding role for the teacher. This study affirms the view that teaching is a complex enterprise where decisions must be made for which no predetermined formula exists.

One of the most powerful affirmations of this unit for the teachers was the displays of insightful student thinking. Although the situations were challenging, the richness of student work and the frequent insights they demonstrated encouraged the teachers to "expect the unexpected" and to persist with the aim of making mathematics meaningful and relevant for their students.

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